

Conductance oscillations in mesoscopic rings: microscopic versus macroscopic picture

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The phase of Aharonov-Bohm oscillations in mesoscopic metal rings in the presence of a magnetic field can be modulated by application of a DC-bias current I_{DC} . We address the question of how a variation of I_{DC} and hence of the microscopic phases of the electronic wave functions results in the macroscopic phase of the conductance oscillations. Whereas the first one can be varied continuously the latter has to be quantized for a ring in two-wire configuration by virtue of the Onsager symmetry relations. We observe a correlation between a phase flip by $\pm\pi$ and the amplitude of the oscillations.

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I. INTRODUCTION

In mesoscopic rings exposed to a perpendicular magnetic field B the interference of partial waves of electrons propagating phase-coherently in opposite directions leads to oscillations of the magnetoconductance with a fundamental period $B_{per} = \phi_0/A$, where $\phi_0 = h/e$ is the flux quantum and A the area of the ring [1]. In rings with finite width of the arms the interference of waves within one arm of the ring results in a modulation of the amplitude and the phase of the oscillations [2]:

$$\Delta G = g(B, E) \cos\left(2\pi \frac{BA}{\phi_0} + \varphi(B, E)\right) \quad (1)$$

The oscillations of ΔG with respect to B are usually termed Aharonov-Bohm (AB) effect although strictly speaking the AB effect in its original meaning refers to a phase shift of electron waves by a vector potential only [3]. The amplitude g and the phase φ are sample specific as they depend on the microscopic arrangement of the scattering centres in the ring. Both quantities are random functions of the magnetic field B and the energy E of the electrons. The typical scales in B and E for a variation of g and φ are the correlation field B_c and the Thouless energy $E_c \approx hD/L^2$, where D is the diffusion constant and L the sample length.

In mesoscopic devices in general, the symmetry of the magnetoconductance upon magnetic-field reversal depends on the sample geometry [4]. In a two-wire configuration the current and voltage leads branch outside the phase-coherent region of the electrons and the conductance G is symmetric with respect to the magnetic field B , $G(B) = G(-B)$. This symmetry relation does not hold for a four-wire configuration with a bifurcation of the voltage and current leads within the phase-coherent region. This behaviour is based on the fundamental Onsager relations which are a consequence of time-reversal symmetry. Onsager succeeded in deriving general reciprocal relations from the principle of microscopic reversibility [5]. The application of the Onsager relations to

the electrical conductance as a macroscopic quantity was discussed by Casimir [6]. The necessity to distinguish between mesoscopic two-wire and four-wire configurations was demonstrated theoretically by Büttiker [4].

These symmetry relations have been confirmed experimentally in many experiments, e.g. with mesoscopic rings [7]. For a ring in two-wire configuration the symmetry condition does not allow arbitrary values of the phase of the AB oscillations. On the other hand, in a microscopic picture this phase is determined by the arbitrary (but fixed for a given field) difference between electrons travelling through the two arms which in turn depends on the phase an electron accumulates at the scattering centres in the metal ring during its diffusive motion. In our experiment we address the question of how the quantization condition of the macroscopically observable phase of the conductance oscillations is fulfilled when the microscopic electronic phase is varied continuously. By application of a DC-bias current I_{DC} we generate a non-equilibrium energy distribution of the electrons in the ring since no energy relaxation occurs in the phase-coherent region. The energy of the electrons contributing to the charge transport and hence also the phase of the microscopic electronic wave function can be modified by a variation of I_{DC} . We analyze the reaction of the phase of the AB oscillations on this continuous variation on the microscopic level by monitoring the cross-correlation for magnetoconductance traces at different I_{DC} on the macroscopic level.

II. EXPERIMENTAL

The samples were prepared by electron-beam lithography and evaporation of Cu or Ag on a Si substrate followed by a lift-off process. The differential resistance dV/dI was measured at 19 Hz with a superimposed DC-bias current I_{DC} with the sample mounted in the mixing chamber of a dilution refrigerator with a base temperature of 20 mK.

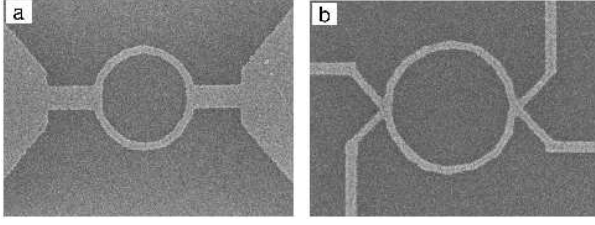


FIG. 1. Electron microscope photographs of a ring in a) two-wire and b) four-wire configuration. The rings have a diameter of 1 μm and a linewidth of a) 80 nm and b) 60 nm. The thickness of the Cu film is 15 nm.

Figure 1 shows electron-microscope photographs of two rings. In the two-wire configuration the voltage and current leads branch at a distance of 4 μm from the ring (not shown in the photograph) which is more than the phase-coherence length of $\approx 1 \mu\text{m}$. In the four-wire configuration the bifurcation is directly at the ring.

III. RESULTS AND DISCUSSION

Figure 2 displays the magnetoconductance of a ring in two-wire configuration. The magnetic-field axis was corrected by an offset of 4 mT (which was the same for all measurements) due to the hysteresis of the superconducting magnet. The AB oscillations with a period of 5.2 mT are clearly visible and it is evident that their amplitude and phase vary with I_{DC} . At $I_{DC} = 15.2 \mu\text{A}$ the amplitude has a minimum and the phase has flipped by π . In a separate publication [8] we showed that the average amplitude of the oscillations increases with increasing I_{DC} . For the evaluation of the phase shift we used this effect to measure oscillations with larger amplitude at large I_{DC} and hence to increase the signal-to-noise ratio. Therefore the data presented here were measured at a current of several μA . The AB oscillations of a ring in four-wire configuration (not shown) are also shifted with I_{DC} , but they are not symmetric upon reversal of B , in agreement with theoretical prediction [4] and previous experimental observation [7].

The average phase shift in the whole investigated magnetic-field range is analyzed quantitatively by calculating the cross-correlation function (CCF) between the conductance oscillations measured at currents I_1 and I_2 :

$$C(I_1, I_2, \Delta B) = \int \Delta G_{I_1}(B) \Delta G_{I_2}(B + \Delta B) dB$$

The CCF of two periodic functions with the same period is again a periodic function. A shift of δB between these functions manifests itself in a shift of the maxima of $C(\Delta B)$ by δB .

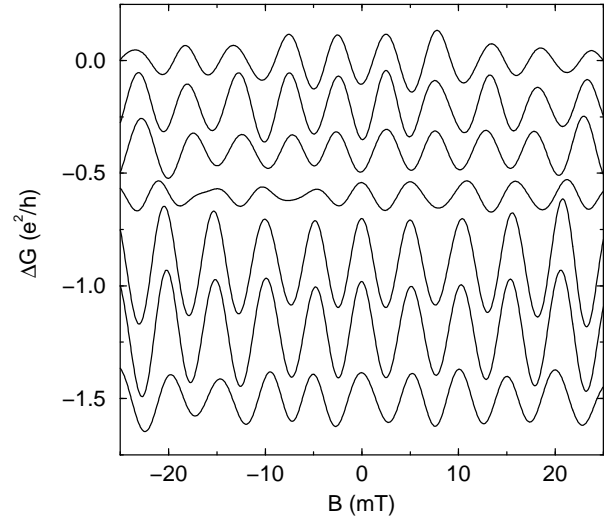


FIG. 2. Conductance ΔG of a ring in two-wire configuration vs. magnetic field B at a temperature $T = 90 \text{ mK}$ and currents $I_{DC} = 14.0, 14.4, 14.8, \dots, 16.4 \mu\text{A}$ (from top to bottom). The data were digitally Fourier filtered. Only frequencies corresponding to a period range from 3 mT to 10 mT were taken into account. The data are offset vertically for clarity.

Figure 3a shows the evaluation of the oscillations displayed in fig. 2 for a ring in the two-wire configuration. The CCF's are calculated for $\Delta G(B)$ taken at different $I_2 = I_{DC}$, each time with reference to $\Delta G(B)$ at $I_1 = 14.0 \mu\text{A}$. It can be seen that the oscillations are either in phase or shifted by $\delta B = B_{per}/2$. For comparison the evaluation for a ring in four-wire configuration is displayed in fig. 3b. Here the shift δB between the oscillations is arbitrary.

The phase shift of the oscillations $\delta\varphi$ is related to δB by the relation $\delta\varphi = 2\pi\delta B/B_{per}$. The observed shift of the CCF's results in $\delta\varphi = 0$ or $\delta\varphi = \pi$ for a two-wire configuration (cf. fig. 4a) whereas $\delta\varphi$ is arbitrary for a four-wire configuration. This behaviour demonstrates convincingly

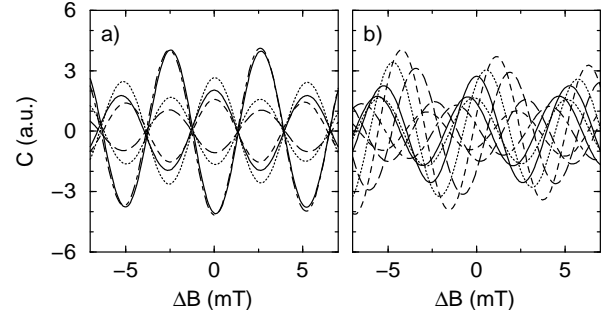


FIG. 3. a) Cross-correlation functions $C(I_1, I_2, \Delta B)$ for $I_1 = 14.0 \mu\text{A}$ and different $I_2 = I_{DC}$ vs. ΔB for a ring in two-wire configuration. The magnetoconductance is displayed in fig. 2 and was evaluated in a magnetic-field range from -30 mT to 30 mT . b) Cross-correlation functions for a ring in four-wire configuration for comparison.

the expected symmetry relations as the magnetoconductance of a sample in two-wire configuration has to be symmetric upon reversal of B . This restricts the phase φ in eq. (1) to either 0 or π whereas there is no constraint for a four-wire configuration. A quantitative analysis performed over a wide range of I_{DC} revealed that the typical current scale for phase flips corresponds to the Thouless energy E_c , in agreement with eq. (1).

From a simple point of view the AB oscillations arise from the interference of an electron which splits into two partial waves at one side of the ring. These waves traverse the ring in opposite arms and recombine at the other side. The interference is determined by the phase difference of the wave functions. From this simple argument the phase of the wave functions should be the same at $B = 0$ provided that the arms have equal lengths. Hence at $B = 0$ the interference should be constructive and the conductance have a maximum.

However, the phase of the electronic wave function also depends on the configuration of the scattering centres. For this reason the phase will usually not be the same in both arms of the ring, so that on the one hand at $B = 0$ an arbitrary interference between the partial waves might be possible. Indeed it was demonstrated that the AB oscillations average to zero when the measurements are taken of a series of rings [9]. On the other hand the symmetry relations for the magnetoconductance require that for a ring in a two-wire configuration the conductance at $B = 0$ has either a maximum or a minimum. This means that the interference has to be either constructive or destructive.

The key to the resolution of these seemingly contradictory statements is that the above argument considering only two wave functions is too simple. For the correct calculation of the conductance all transmitted and reflected partial waves have to be taken into account. However, an interesting question arises when we vary the phases of the electronic wave functions continuously by a modification of I_{DC} : what is the reaction of the system to this continuous variation of the phases of the wave functions under the condition that the macroscopically observable phase of the conduction oscillations is quantized?

For this purpose the phase and the amplitude of the oscillations are displayed in a common graph in fig. 4. The data were extracted from a series of magnetoconductance traces (some of them are shown in fig. 2) measured on a ring in two-wire configuration. It can be seen that both quantities are a function of I_{DC} . There is a definite correlation between the variation of the amplitude and the phase: at each phase flip by π there is a minimum of the oscillation amplitude. Hence no abrupt flip of the conductance oscillations occurs, but every variation of the phase by $\pm\pi$ is accompanied by a continuous decrease and subsequent increase of the oscillation amplitude. Superimposed on the strong fluctuations of the oscillation amplitude an increase of ΔG_{rms} is observed. Averaging

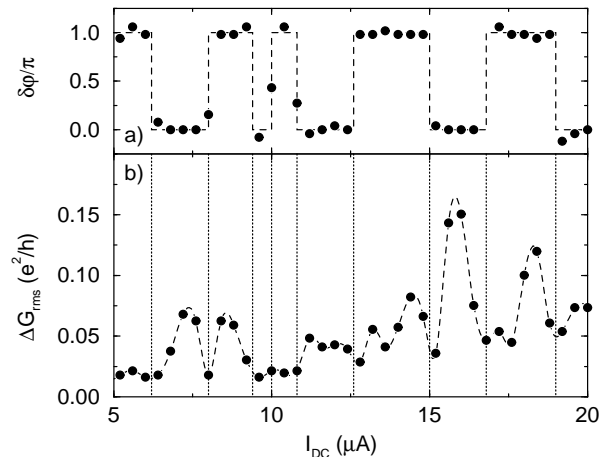


FIG. 4. a) Phase shift referred to $I_{DC} = 7.6 \mu\text{A}$ and b) rms-amplitude of the conductance oscillations vs. I_{DC} . Some of the magnetoconductance traces are displayed in fig. 2. The data were measured at a ring in two-wire configuration and evaluated in a magnetic-field range from -30 mT to 30 mT . The dashed lines are guides to the eye. The dotted vertical lines mark the positions of a phase flip by $\pm\pi$.

over a larger field interval than just $[-30 \text{ mT}, 30 \text{ mT}]$ yields $\Delta G_{rms} \sim \sqrt{I_{DC}}$, in agreement with the prediction for the conductance fluctuations in singly connected mesoscopic samples [8].

In conclusion, although the microscopic phase of the electronic wave function is varied continuously by a modification of I_{DC} the macroscopic phase of the conductance oscillations varies in a quantized manner. However, there is no abrupt change in the magnetoconductance. Rather, the macroscopic phase flip is accommodated by a rearrangement of the individual electron phases to produce an interference pattern leading to a minimum of the oscillation amplitude. This continuous variation of the interference on the microscopic level is directly visible (cf. fig. 2) as a slight shift of the oscillation frequency whenever a phase flip occurs. In a recent experiment on semiconductor rings in two-wire configuration with a few electron transmission channels the phase of the oscillations could be modified by variation of a gate voltage [10]. A phase flip by π of the fundamental oscillation was accompanied by the occurrence of a dominating higher harmonic with $h/2e$ periodicity. In our experiment on diffusive rings with many electron transmission channels we observe oscillations with only the fundamental h/e periodicity whose suppression at macroscopic phase flips indicates the rearrangement at the microscopic level.

IV. ACKNOWLEDGEMENTS

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